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The theory of induced voltage electromagnetic flowmeters

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The performance of an electromagnetic flowmeter head is assessed in terms of a weight vector **W** such that the output voltage $\propto \int \mathbf{v} \cdot \mathbf{W}\, d\tau$, where **v** is the velocity and τ the flowmeter volume. The condition curl $\mathbf{W} = 0$ with $\mathbf{W} \rightarrow 0$ at ∞ is shown to be necessary and sufficient for the velocity to depend only on the flow rate and not on the flow pattern. A class of such 'ideal' meters is described. It is shown that meters with point electrodes can never be ideal but may, with considerable complication of the magnetic field, be made immune to asymmetric velocity-profile variations if the flow is rectilinear.

1. Introduction

Electromagnetic flow measurement is at present applied to three groups of liquids: liquid metals, water-based industrial liquids, and blood. Shercliff (1962) has described many of its features. This paper is mainly concerned with flow in pipes, although some of the results can be applied to other geometries. The aim is to examine the conditions under which the output signal of a flowmeter is independent of the velocity distribution and thus always proportional to flow rate.

The sensitivity of a flowmeter with specified electrodes and magnetic field may be defined as

$$\text{the induced voltage between the electrodes, the flow rate} \\ \times \text{the magnetic field at a fixed position.}$$

This definition ensures that a *pro-rata* increase in the magnetic field at every point due to an increase of current in the field windings does not alter the sensitivity. The latter is inversely proportional to the linear scale of the flowmeter. The sensitivity in general depends on the spatial distribution of the magnetic field **B**, the distribution of velocity **v** and the electrode arrangement. The aim of the flowmeter designer is to arrange the magnetic field and electrodes so that this dependence on **v** disappears and the output signal is proportional to the flow rate regardless of the distribution of **v**. The often difficult task of predicting **v** does not arise. Liquid metal applications where the induced currents are large enough so that **B** also depends on **v** are not considered.

It has been usual to assume that in flowmeters of circular cross-section the flow is rectilinear with an axisymmetric profile. If the profile is not specified, it may be

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termed asymmetric. In practice, provided a sufficient length of straight pipe is incorporated upstream of the flowmeter and the flow is steady, an axisymmetric and convex profile may be expected.

It is well known that the sensitivity of flowmeters employing a uniform transverse magnetic field, diametrically opposed electrodes and a non-conducting pipe is constant only when the velocity profile is axisymmetric. Most designs have assumed these features. Even when the profile is axisymmetric the sensitivity depends upon its form if the magnetic field is non-uniform. Until recently a compromise has been required between the inconveniently long uniform magnetic fields with which axisymmetric flow may be correctly measured and conveniently short fields which do not permit this.

However, Rummel & Ketelsen (1966) have shown that deliberately short non-uniform fields produced by specially shaped coils can improve the performance of a flowmeter by rendering it less sensitive to asymmetric velocity profiles. Their work is an extension of the idea of a weight function first introduced by Shercliff (1954, 1962), which indicates how each part of the flow contributes to the signal.

Clark & Wyatt (1967) have tested a circular meter with point electrodes and a range of short rectilinear air-cored coils approximately one diameter long, and measured its sensitivity as the Reynolds number was increased and the flow changed from laminar to turbulent. Their results show that there is a coil for which the sensitivity does not change with Reynolds number.

Kanai (private communication) has computed the sensitivity for axisymmetric and asymmetric profiles in flowmeters with air-cored coils of various shapes and compared the results with experiment. Agreement is mostly within 3 %.

In this paper: (1) The idea of a weight function is put on a more rigorous basis, by introducing a weight vector **W**. (2) The condition on **W** for an ideal meter is given, and a class of such meters described. (3) It is proved that flowmeters with point electrodes can never be ideal. (4) Weight functions that are suitable for restricted types of flow pattern are derived from **W** and conditions given on these weight functions to ensure that the flowmeter is ideal with respect to these flow patterns. (5) The possibility of achieving the conditions in (4) is examined in the case of flowmeters with point electrodes.

2. The weight vector W

The existence of **W** stems essentially from the reciprocal relations that apply in electrical networks or continuous media. We consider how the motion of an electrically conducting liquid with velocity $\mathbf{v}(\mathbf{r})$ in a magnetic field $\mathbf{B}(\mathbf{r})$ contributes to the voltage between two electrodes in the liquid. If the conductivity is uniform the voltage between the electrodes can be found from the reciprocal relations between charges and potentials in two different states of a system, together with the flowmeter equation (Shercliff 1962, p. 13)

$$\nabla^2 U = \operatorname{div}(\mathbf{v} \times \mathbf{B}), \tag{1}$$

where U is the induced electrical potential, and $\operatorname{div}(\mathbf{v} \times \mathbf{B})$ is treated as a charge distribution. If the conductivity is anistropic and non-uniform, the voltage can be found from the equivalent electrical network and the reciprocal relations between current and voltages, by treating the components of $\mathbf{v} \times \mathbf{B}$ as voltages imposed between the nodes of the network. These were the methods originally used, but the following general derivation of $\mathbf{W}$ is more rigorous.

$\mathbf{j}$ and $\mathbf{E}$ are current and electric fields respectively. Ohm's law is taken in the form $j_i = \sigma_{ij} E_j$ where $\sigma_{ij}(\mathbf{r})$ is a positive-definite symmetric tensor† that may depend on position. In certain situations, such as blood flow with orientation of the red blood cells (Dennis & Wyatt 1969) the anisotropy and variation of the conductivity may be important, though in most cases it is not. Boundary conditions on the wall of the flowmeter do not have to be taken into account explicitly since they can be achieved by abrupt (quasi-continuous) variation of σ_{ij}.

Let U^m and $\mathbf{j}^m$ be the potential and current induced by the motion and U and $\mathbf{j}$ those set up when unit current is passed between the electrodes with no motion. Then Ohm's law gives

$$j_i^m = \sigma_{ij}[-\partial U^m/\partial x_j + (\mathbf{v} \times \mathbf{B})_j], \tag{2}$$

$$j_i = -\sigma_{ij}\, \partial U/\partial x_j. \tag{3}$$

Let S_1 and S_2 be surfaces around the electrodes and τ, for the present, the whole of space. Then because $\operatorname{div} \mathbf{j}^m = \operatorname{div} \mathbf{j} = 0$ and $\mathbf{j}^m$ and $\mathbf{j} = O(1/R^3)$ for large R, Gauss's theorem gives

$$\int_{S_1+S_2} (U^m \mathbf{j} - U \mathbf{j}^m) . d\mathbf{S} = \int (\nabla U^m . \mathbf{j} - \nabla U . \mathbf{j}^m)\, d\tau. \tag{4}$$

If no current is drawn from the electrodes during operation as a flowmeter then

$$\int_{S_1} \mathbf{j}^m . d\mathbf{S} = \int_{S_2} \mathbf{j}^m . d\mathbf{S} = 0$$

and
$$\int_{S_1} \mathbf{j} . d\mathbf{S} = -\int_{S_2} \mathbf{j} . d\mathbf{S} = 1.$$

Also, U^m, U are constant on the electrodes (which are taken to have infinite conductivity) and the voltage difference $U_1^m - U_2^m$ between them is given by the left-hand side of (4). Substituting (2) and (3) into the right-hand side of (4) we find

$$U_1^m - U_2^m = \int \left[-\frac{\partial U^m}{\partial x_i} \sigma_{ij} \frac{\partial U}{\partial x_j} + \frac{\partial U}{\partial x_i} \sigma_{ij} \frac{\partial U^m}{\partial x_j} - \frac{\partial U}{\partial x_i} \sigma_{ij} (\mathbf{v} \times \mathbf{B})_j \right] d\tau$$

and using the symmetry of σ_{ij} and (2) this becomes

$$U_1^m - U_2^m = -\int (\mathbf{v} \times \mathbf{B}) . \mathbf{j}\, d\tau,$$

which may be written
$$U_1^m - U_2^m = \int \mathbf{v} . \mathbf{W}\, d\tau, \tag{5}$$

where $\mathbf{W} = \mathbf{B} \times \mathbf{j}$ is called the weight vector, an extension of Shercliff's (1954 or 1962, p. 29) weight function. τ may now be restricted to the moving fluid because $\mathbf{v} = 0$ outside it. $\mathbf{j}$ is determined by the electrode shape and electrical conditions on the flowmeter wall (and by the conductivity distribution if non-uniform) and

† I am indebted to one of the referees for pointing out that the physical reasons for the symmetry of σ_{ij} are given in Landau & Lifshitz (1960, chapter 3).

will be called the virtual current, virtual to distinguish it from the actual currents that flow when the flowmeter is in operation. In the case where the induced magnetic field is small and the conductivity uniform (this we shall assume from now on unless otherwise stated) curl $\mathbf{B}$ and curl $\mathbf{j}$ = curl $\sigma\mathbf{E} = 0$. We may set $\mathbf{B} = \nabla F$ and $\mathbf{j} = \nabla G$ giving

$$\mathbf{W} = \nabla F \times \nabla G, \tag{6}$$

where both F and G are solutions of Laplace's equation. In this case $\operatorname{div}\mathbf{W} = 0$ automatically.

Condition on **W** *for an ideal meter*

The virtual current $\mathbf{j}$ depends on the electrode shape and the flowmeter geometry. The aim of the flowmeter designer is to arrange $\mathbf{B}$ and $\mathbf{j}$ so that the sensitivity is independent of the flow pattern. The signal will then be proportional to flow rate irrespective of the velocity distribution, and the meter will be ideal. If the fluid is incompressible ($\operatorname{div}\mathbf{v} = 0$) and if the electrodes or magnetic field are confined ($\mathbf{W} \to 0$ upstream and downstream) then the necessary and sufficient condition for the signal to depend only on the flow rate is curl $\mathbf{W} = 0$.†

Sufficiency. If curl $\mathbf{W} = 0$, then $\mathbf{W} = \nabla t$ for some function $t(\mathbf{r})$. Also, where $\mathbf{W} \to 0$ away from the meter $t \to$ constant, but may take different values t_u and t_d upstream and downstream of the flowmeter head. The signal is (from (5))

$$\int \mathbf{v}.\nabla t\, d\tau$$

or, since $\operatorname{div}\mathbf{v} = 0$

$$\int_S t\mathbf{v}.d S,$$

where the surface S consists of the flowmeter wall and two surfaces spanning the meter cross-section, one far upstream (S_u) and the other far downstream (S_d). Since $\mathbf{v}.d\mathbf{S} = 0$ on the wall and

$$-\int_{S_u} \mathbf{v}.d\mathbf{S} = \int_{S_d} \mathbf{v}.d\mathbf{S} = Q, \quad \text{the flow rate,}$$

the signal is

$$(t_d - t_u)\,Q, \tag{7}$$

which is proportional to Q since t_u and t_d are fixed.

Necessity. If curl $\mathbf{W} \neq 0$, there is a loop C around which

$$\oint_C \mathbf{W}.d\mathbf{r} \neq 0.$$

Consider the flow pattern in which the fluid is stationary everywhere except along a tube of small cross-section A in the direction $d\mathbf{r}$. Then the volumetric flow along this tube must be constant, since $\operatorname{div}\mathbf{v} = 0$. Let it be q. This velocity pattern, which produces no net flow rate, gives rise to a signal

$$q\oint_C \mathbf{W}.d\mathbf{r},$$

where $\mathbf{v}\,d\tau = q\,d\mathbf{r}$ has been substituted in (5). Thus $\oint_C \mathbf{W}.d\mathbf{r}$ must be zero around every possible loop C, i.e. by Stokes's theorem curl $\mathbf{W} = 0$.

† This result and (5) were reported by Shercliff (1967*b*): (5) was also mentioned in Shercliff (1967*a*).

3. Types of ideal flowmeter

Since $\mathbf{W} = \nabla F \times \nabla G$ we seek solutions of Laplace's equation, F and G, such that

$$\left.\begin{aligned} \operatorname{curl}(\nabla F \times \nabla G) &= 0 \\ \text{or alternatively} \qquad \nabla F \times \nabla G &= \nabla t, \end{aligned}\right\} \tag{8}$$

where t itself will be Laplacian since div $\mathbf{W}$ is automatically zero. Only two sets of F and G have been found that satisfy (8) and the existence of others has not been established. The first set is of the type $F = \alpha x^2 + \beta y^2 + \gamma z^2$, $G = \alpha' x^2 + \beta' y^2 + \gamma' z^2$ and $t \propto xyz$. It is not suitable since $\nabla t \to 0$ at ∞.

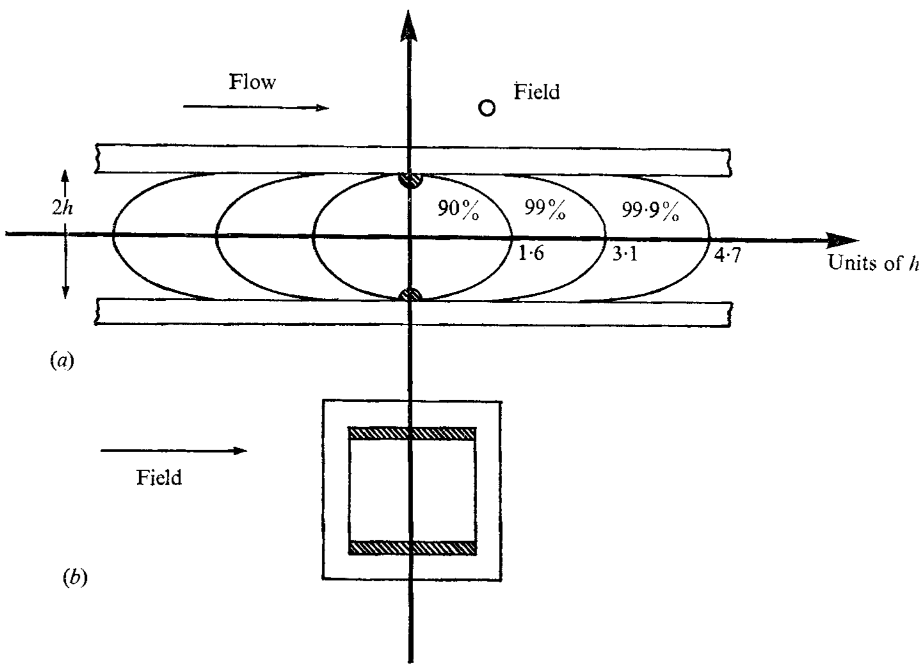


FIGURE 1. A simple ideal flowmeter with a uniform field and insulating walls perpendicular to the field, showing virtual current lines. (*a*) Section perpendicular to field, (*b*) cross-section.

The second set is very simple and provides a class of ideal flowmeters that have some very remarkable properties. In these meters either ∇F or ∇G must be constant, and the other lie in planes perpendicular to it. The case of constant ∇F (constant magnetic field B_0) only will be considered. The dual set of flowmeters with constant ∇G are not so practicable.

The requirement that ∇G is perpendicular to the magnetic field B_0 means that the flow channel must be a cylinder with generators parallel to the field and with insulating walls at either end perpendicular to the field. A simple form of meter is shown in figure 1. A rectangular flowmeter with long electrodes is a special case. One was used by Arnold (1951) though it was not then realized that it was ideal.

With G, a plane solution of Laplace's equation, we may associate a stream function ψ_G. Then

$$\mathbf{W} = B_0 \nabla \psi_G,$$

$B_0 \psi_G$ being the function t of (7).

The condition that unit virtual current emerges from unit width of electrode gives, in the notation of (7)

$$t_u - t_d = B_0$$

and substituting this in (7) we find that the output signal is

$$B_0 \times \text{flow rate per unit width.}$$

The sensitivity of these meters† is thus independent of their shape (provided they are of constant width) and depends only on the magnetic field. Indeed the shape may be changed while the flowmeter is operating, and it may even be used as a valve at the same time if movable flaps or arms are provided which are generated by lines parallel to the field. There is no restriction on the number of entrances and exits and each may have its own valve and set of electrodes. Such a device could be used as a multiple metering valve (figure 2).

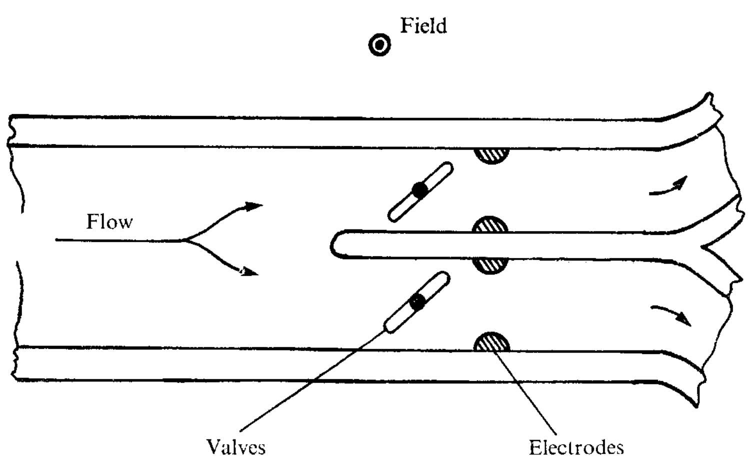


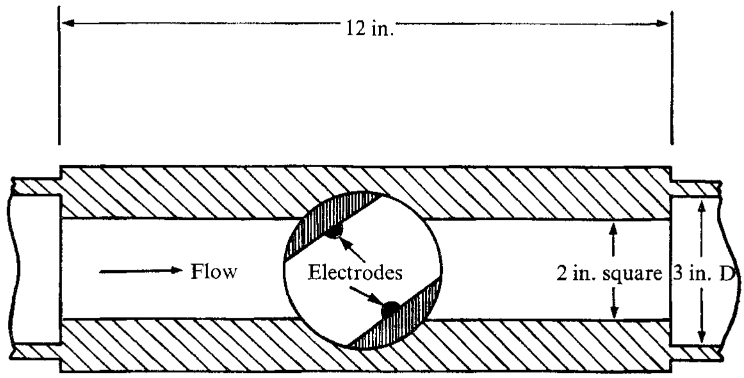
FIGURE 2. An example of a valve flowmeter with multiple exits, section perpendicular to field.

The virtual current lines for the flow meter in figure 1 are taken from Bewley (1963, figure 37). Similar details for flowmeters with large electrodes are given, in a different context, by Rossow (1960). Flux plotting is probably a sufficiently accurate means of obtaining virtual current lines for more complicated flowmeters, since the aim is to establish the region containing a specified proportion of the virtual current, say 99 %, and then to ensure that the magnetic field is uniform there. The variation in sensitivity to be expected from the non-uniformity of the field in the remaining virtual current region will then be $O(1\ \%)$. Since most fields become non-uniform gradually, with most flow patterns the sensitivity would vary by much less than 1 %.

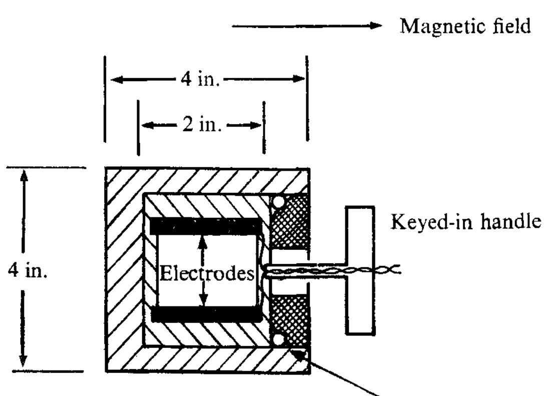
† This class of meters, where **B** is uniform and the boundary conditions invariant in the **B** direction, may also be proved ideal by integrating (1) and the condition div $\mathbf{v} = 0$ in the direction of **B** and noticing that curl $(\mathbf{v}' \times \mathbf{B}) = 0$ where $\mathbf{v}'$ is the resulting integral of **v**. If **v** were invariant in the **B** direction then curl $(\mathbf{v} \times \mathbf{B}) = 0$ and the conductivity would be immaterial.

An experimental flowmeter

Figure 3 shows an experimental ideal meter. It is made of Perspex with metal electrodes of half circular cross-section mounted on a stop cock which can be rotated. This acts as a valve as well as a flowmeter, also creating eddies of a complicated and uncertain nature. This meter was used in an experiment (Bevir



(*a*)



(*b*)

Figure 3. An experimental ideal flowmeter, (*a*) section perpendicular to the magnetic field, (*b*) section perpendicular to the flow.

1969) to verify the above theory by showing that the sensitivity is independent of both stop-cock angle and the corresponding changes in flow pattern.

The magnetic field of about 150 gauss was provided by an external electromagnet driven from the mains and the field uniformity over the central region of the flowmeter was of the order of 1 %. The output signal, about 2 mV r.m.s., was stepped up by using a 1:10 transformer and fed into a Kent Veriflux Mark II converter. This produces a 0–10 mA d.c. current proportional to the in-phase output signal divided by the magnet current. The d.c. current was passed through a 500 Ω stable resistance and the resulting voltage, smoothed by 15,000 μF in parallel with the resistance, was measured on a digital voltmeter. Variations in magnet current were kept within 1 %, which alters the zero by 0·05 %. The system also largely rejected quadrature voltages. As the stop cock was rotated there were variations in quadrature voltage of up to 40 % of flow voltage, and these caused variations in the zero reading on the voltmeter of up to 1·2 %. However, for a given stop-cock position the quadrature was nearly constant and the stability of the zero flow and full flow signals was within ± 0·1 % of the full

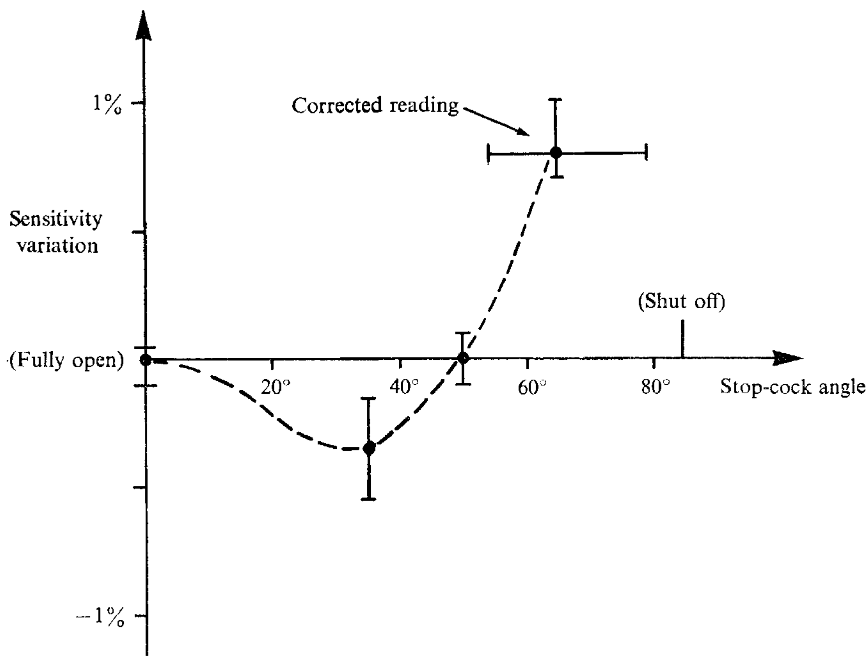


FIGURE 4. Valve flowmeter sensitivity variation with stop-cock position.

flow reading. The mean flow speed was 3 m/s. The flow was also measured using a Venturi meter with a 100 cm water manometer which could be read to ± 0·1 cm. This corresponded to ± 0·1 % of the indicated flow. Temperature effects were never greater than 0·2 % under the conditions of the experiment and usually negligible. The two flowmeters could thus be compared with a repeatability of ± 0·3 % as the results in figure 4 confirm. The Venturi calibration did not affect the first three readings since nearly the same flow rate was used for each run. The fourth reading was obtained with the stop cock almost shut and a manometer reading of 50 cm rather than 90 cm. A correction factor for the reading was computed from the British Standard calibration curves for the Venturi. It was exactly unity, since the Reynolds number correction balanced the temperature correction. The results are given relative to those obtained when the stop cock was fully open.

The results in figure 4 confirm the ideal behaviour of the meter in this flow situation. The small but detectable variations of sensitivity with stop-cock angle are probably due to the non-uniformity of the magnetic field. Though a direct comparison with uniform field circular flowmeters using point electrodes is difficult because of the different geometries, similar eddies with reverse flow near the electrodes can cause sensitivity changes of order 100 % in a uniform magnetic field (Shercliff 1955).

4. Flowmeters with point electrodes

If R is the distance from a point electrode and d a typical dimension of the flowmeter cross-section, e.g. a radius of curvature of the wall at the electrode, then for $R \ll d$ and $\gg$ the electrode dimension,

$$G = -\frac{1}{4\pi R} + O\left[\frac{\log R}{d}\right]. \tag{9}$$

In the appendix it is assumed that near the electrodes $G \sim 1/R$ and it is shown that for this G there is no non-trivial magnetic field $\mathbf{B}$ that satisfies curl $\mathbf{W} = 0$. This proves that flowmeters with point electrodes, which are of great practical importance, cannot be made ideal.

Flow type	Assumption on $\mathbf{v}$	Condition on $\mathbf{W}$
(1) Unspecified	$\operatorname{div} \mathbf{v} = 0$	$\operatorname{Curl} \mathbf{W} = 0$
(2) Ill-founded	$\mathbf{v} = [0, 0, v(x, y, z)]$	$W_z =$ function (z)
(3) Asymmetric Rectilinear	$\mathbf{v} = [0, 0, v(x, y)]$	$\overline{W}(x, y) = \int_{-\infty}^{\infty} W_z dz =$ constant
(4) Axisymmetric Rectilinear	$\mathbf{v} = [0, 0, v(r)]$	$\overline{W}'(r) = \frac{1}{2\pi}\int_0^{2\pi} \overline{W} d\theta =$ constant

TABLE 1. Flow pattern assumptions and conditions on $\mathbf{W}$

Since such flowmeters cannot be ideal when the flow pattern is unspecified some restrictive assumptions may be made about the flow. These are given in order of increasing restriction in table 1 for straight flowmeters of constant cross-section. The notation is in figure 5. Corresponding to each assumption about the flow there is a necessary and sufficient condition on the relevant component of $\mathbf{W}$ to make the flowmeter ideal for the type of flow assumed. Condition (2) is ill-founded since the assumptions $v_x = v_y = 0$ together with $\operatorname{div} \mathbf{v} = 0$ imply $\partial v_z/\partial z = 0$. Nevertheless, it is thought to have been used as the basis of some designs. If the necessary condition on $\mathbf{W}$ for this case can be achieved, which is doubtful, then both this flowmeter and that based on condition (3) are suitable for asymmetric profiles not varying in the flow direction and both will be affected by transverse flows.

Swirl, or transverse flow

The contribution of transverse flows (v_x, v_y) is not in general zero, but is so in some cases, due to the design symmetry of many types of flowmeter. If the flowmeter has the very common symmetry in which (figure 5) F is even in z and y and odd in x, whereas G is even in z and x and odd in y then W_z is even in z, whereas W_x and W_y are odd in z. Thus swirl which does not vary along the pipe ($\partial v_x/\partial z = \partial v_y/\partial z = 0$) will not contribute. Examples of this are secondary flow in a slightly curved pipe or secondary flow set up by a velocity profile varying slowly in the z direction. Again, swirl of the form ($v_r = 0, v_\theta(r,z), v_z = 0$) will not contribute whatever the dependence of v_θ on r and z.

Axisymmetric rectilinear flow

We consider here the case of circular flowmeters with axisymmetric profiles and point electrodes, or equivalent schemes in other geometries where the profile depends only on one variable (e.g. flows past a wall where the velocity depends only on distance from the wall). The axisymmetric weight function $\overline{W}'(r)$ (table 1) is well known to be uniform for point† electrodes and a uniform transverse magnetic field. For this reason singularities in $\overline{W}'(r)$ using a non-uniform field,

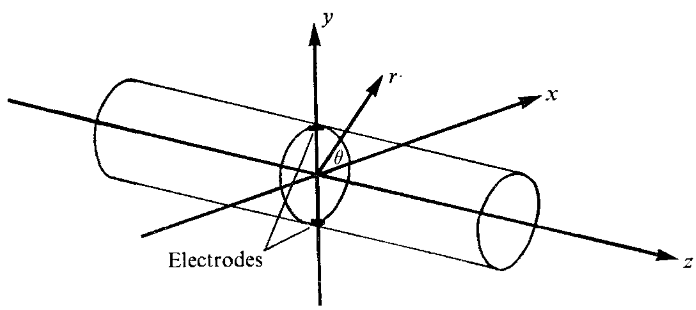


FIGURE 5. Circular flowmeter notation.

locally uniform at the electrodes, are not to be expected. This is confirmed by considering the effect of shortening the field, which is merely to add correction terms to the value of $\overline{W}'$ obtained when the field is uniform (Bevir 1969). It is certainly possible to achieve uniform $\overline{W}'(r)$, although there is more than one way of doing this in practice.

Asymmetric rectilinear flow

It is not easy, using point electrodes, to make the asymmetric weight function $\overline{W}(x,y)$ uniform. It is impossible to do so if a two-dimensional non-uniform magnetic field is used (Bevir 1969). The question is whether the extra freedom gained by allowing the magnetic field to vary three dimensionally will make it possible. In long flowmeter theory the transverse magnetic field along a radius completely specifies both it and the axisymmetric weight function $\overline{W}'(r)$ (Baker 1968; Bevir 1969). We may plausibly expect this situation to extend to short flowmeters and thus to be able to arrange the distribution of the normal component of field, or its potential F, over the surface to produce a specified distribution of the asymmetric weight function $\overline{W}(x,y)$. It now becomes necessary to make the magnetic field zero at point electrodes. There is only one non-singular

† This is true for any shape of electrode if the field is uniform, because $W_z = B_0 \partial G/\partial y$ is then a solution of Laplace's equation, $\overline{W}$ is a plane solution and by the mean value theorem $\overline{W}'$ is uniform.

magnetic field which is zero at the electrode, which makes $\overline{W}$ finite and which has the necessary symmetry (see appendix). It happens to be two dimensional, giving the same singularity as in the corresponding long flowmeter case (Bevir 1969, figure 2*a*): namely that locally $\overline{W} = \cos 2\phi$. Locally uniform $\overline{W}$ can only be achieved by bringing coils or pole pieces right up to the electrode (see appendix). Electrodes of finite size help (Bevir 1969, p. 12) but whether $\overline{W}$ can be made sufficiently uniform with a practical magnetic field is not known.

5. Discussion

It is possible using certain shapes and electrodes to design electromagnetic flowmeters the sensitivity of which is independent of the flow pattern. Conventional flowmeters with point electrodes cannot have this property, though they can with difficulty be made immune to variations in the profile of a rectilinear flow.

Circular flowmeters may be made more nearly ideal by the use of a uniform field with either line electrodes transverse to the flow or with large electrodes. Though either type of electrode yields an ideal meter if the meter is rectangular, it appears at present that in a circular meter line electrodes are to be preferred. They are more convenient in practice and the singularity of the virtual current at the edges $\sim \log z'$, where z' is a local complex variable with origin at the end of the electrode, as opposed to $1/z'$, for point electrodes and $1/\sqrt{z'}$ for large electrodes.

The line electrodes have the weakest singularity, indeed in a strictly transverse field parallel to the electrode $\overline{W}$ would be $\theta = \tan^{-1}(y/x)$ and thus always finite. Though in practice this is unlikely to be the case at the edge of curved electrodes the singularity of $\overline{W}$ would still be much weaker than in the other cases. The weight functions for curved line and large electrodes in long circular flowmeters are given by Bevir (1969), as well as those for point electrodes. The assessment of approximately ideal meters requires more sophisticated analysis than that given here. For example, the circular meter with transverse line electrodes and a uniform field may be susceptible to transverse flows, which is not so with large electrodes.

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Appendix. The behaviour of W near a point electrode

We first show that using point electrodes no magnetic field can be chosen so that $\operatorname{curl} \mathbf{W} = 0$. Secondly we examine the behaviour of $\overline{W}$ near the electrodes. In both cases the assumption is that

$$G = -1/R,$$

where (R,θ,ϕ), (r,ϕ,z), (x,y,z) are spherical, cylindrical and Cartesian co-ordinates with their origin at the electrode. The z axis is in the flow direction and the x axis perpendicular to the wall.

First, let F be a solution of Laplace's equation. From (6)

$$\mathbf{W} = \frac{1}{R^3}\left[0, \frac{-1}{\sin\theta}\frac{\partial F}{\partial\phi}, \frac{\partial F}{\partial\theta}\right] \tag{A 1}$$

and the three components of curl $\mathbf{W} = 0$ lead to

$$\sin\theta\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial F}{\partial\theta}\right) + \frac{\partial^2 F}{\partial\phi^2} = 0, \tag{A 2}$$

$$\frac{\partial}{\partial R}\left(\frac{1}{R^2}\frac{\partial F}{\partial\theta}\right) = 0, \tag{A 3}$$

$$\frac{\partial}{\partial R}\left(\frac{1}{R^2}\frac{\partial F}{\partial\phi}\right) = 0. \tag{A 4}$$

Since $\nabla^2 F = 0$ (A 2) may be written

$$\partial^2(RF)/\partial R^2 = 0; \tag{A 5}$$

(A 3) and (A 4) show that $\quad \partial(F/R^2)/\partial R = f_1(R),$

whence $\quad F = f_2(R) + R^2 f_3(\theta,\phi),$

where f_2 is an arbitrary function of R, $f_1 = d[f_2/R^2]/dR$ and f_3 is an arbitrary function of (θ,ϕ). Equation (A 5) then shows that f_3 must be zero and f_2 be of the form

$$A + BR^{-1}.$$

This magnetic field, apart from having a pole at the electrode, makes $\mathbf{W} = 0$. Thus the only magnetic field that makes curl $\mathbf{W} = 0$ everywhere also makes $\mathbf{W} = 0$ identically.

Secondly,
$$W_z = -W_\theta \sin\theta$$
$$= (1/R^3)\,\partial F/\partial\phi,$$

so that

$$\overline{W}(r,\phi) = \int_{x,\,y\,\text{const.}}^{\infty} W_z\,dz = -\frac{1}{r^2}\int_0^{\pi}\frac{\partial F}{\partial\phi}\sin\theta\,d\theta. \tag{A 6}$$

It is clear $\partial F/\partial\phi$ must be zero at $R = 0$, and F may be expanded in a series of associated Legendre functions. F must be odd in ϕ and even in $\cos\theta$.

This restricts possible choice of F to

$$F = \phi\sum_{n=1}^{\infty} A_n P_{2n}^0(\cos\theta)\,R^{2n} + BR^2 P_2^2(\cos\theta)\sin 2\phi + O(R^3). \tag{A 7}$$

This expansion is valid near the electrode, but not as $R \to \infty$, and A_n, B are constants. The P_{2n}^0 terms produce for $n > 1$ non-zero finite integrals in (A 6) since they converge at the origin and thus make $\overline{W}$ independent of ϕ and r near the electrodes. Legendre functions of fractional order are not considered, though once singular solutions are admitted they could be used. The case $n = 1$ results in $\overline{W} \propto \log r$ and must be suppressed unless this weak singularity can be tolerated. This may be the case since the finite size of the electrode weakens the virtual current nearby. F on the wall is the same as the current stream function for the current sheet required to produce it, and hence gives the shape of the necessary coils. The resulting constant F lines in the first desirable case $n = 2$ with 5 lobes are shown in figure 6. In general there are $2n+1$ lobes.

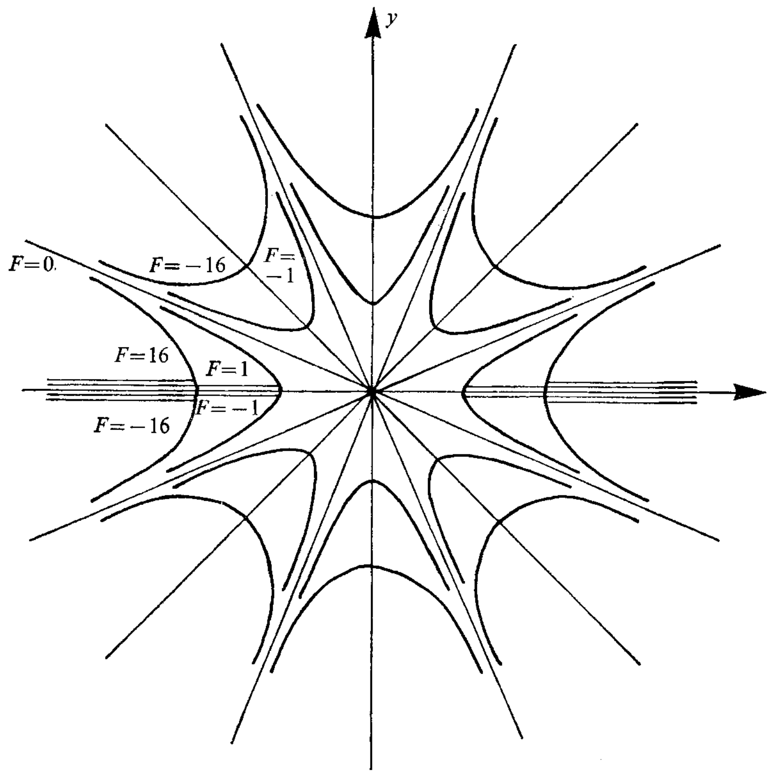

FIGURE 6. Lines of constant F near a point electrode for uniform $\overline{W}$.

The other terms in (A 7) are unwanted fields, the first of which must be suppressed. The terms in R^3 and higher powers of R do not matter since they are zero on the $\theta = 0$, π axis and give zero integrals in (A 6), whereas the first term $R^2P_2^2(\cos\theta)\sin 2\phi$, although zero on this axis, is the plane solution $F \propto xy$ in disguise and gives $\overline{W} \propto \cos 2\phi$ as expected. This field also satisfies $F = 0$ on the flowmeter surface. Therefore specifying F on this surface in such a way that one of the desirable P_{2n}^0 cases is produced does not prevent this field from appearing. It would have to be suppressed in some other way, probably by adjusting the magnetic boundary conditions far from the electrode.

Conclusions

In flowmeters with normal symmetry and no singularity of the field around the electrode the point electrode approximation yields $\overline{W} \propto \cos\phi/r$ if the field is finite. If the field is zero $\overline{W} \propto \cos 2\phi$. If special precautions are taken $\overline{W}$ may be made zero.

If the $F = P_{2n}^0$ fields are produced by pole pieces of the shape and potentials shown in figure 6, then $\overline{W} \propto \log r$ for $n = 1$. $\overline{W} = A + B\cos 2\phi$ for $n > 1$, where

$B \cos 2\phi$ appears unless special precautions are taken. Figure 7 shows some coil shapes deduced from figure 6. These, when surrounded by highly permeable iron, might produce a uniform $\overline{W}$ in a point electrode meter.

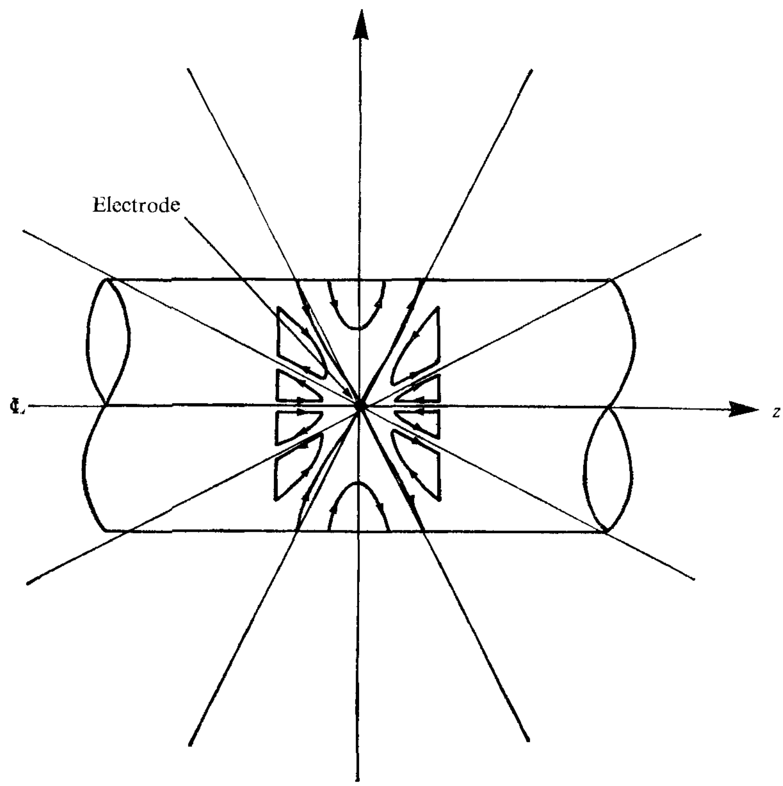

FIGURE 7. Possible shapes of coil surrounding a uniform $\overline{W}$ point electrode flowmeter, with surrounding magnetic circuit removed.

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